

Relaxed Gradient Projection for PCI Assignment in 5G Network

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Abstract—The optimization of Physical Cell Identity (PCI) assignment is a critical challenge in 5G network deployment, as improper assignments can lead to severe network issues, including collisions, confusions, and mod-3 interference. This paper proposes a novel Relaxed Gradient Projection (RGP) method that integrates discrete and continuous optimization techniques to effectively minimize these issues. Unlike conventional discrete approaches such as graph coloring, heuristic methods, and binary quadratic programming (BQP) approaches, RGP reformulates the PCI assignment problem into a continuous optimization problem over the Cartesian product of probability simplexes. A gradient projection algorithm then computes near-optimal assignments, which are subsequently rounded to discrete PCI assignments. Extensive numerical evaluations on real-world 5G network data demonstrate that RGP significantly reduces mod-3 interference while maintaining superior performance in minimizing collisions and confusions. Moreover, RGP scales efficiently to large networks, offering a practical and computationally efficient solution for PCI optimization.

Index Terms—Network Optimization, PCI Assignment, Probability Simplex Relaxation, Gradient Projection

I. INTRODUCTION

The deployment of fifth-generation (5G) mobile networks marks a transformative shift in telecommunications, enabling ultra-high data rates, low latency, and massive device connectivity [1]–[3]. These advancements support critical applications such as autonomous vehicles [4], smart cities [5], and the Internet of Things [6], placing significant demands on network infrastructure [7]. As 5G networks continue to expand, ensuring reliable and high-performance communication requires optimizing key network parameters [8].

Physical Cell Identity (PCI) is one of the most crucial parameters in 5G network configuration, serving as a unique identifier for each cell. It enables mobile devices to recognize primary and neighboring cells, perform cell re-selection, and ensure seamless handovers. However, improper PCI assignments can cause co-frequency interference, collisions, and

confusion, degrading network performance and user experience [9]. As 5G deployments expand, optimizing PCI assignment becomes increasingly critical, particularly in dense urban and suburban environments.

The PCI assignment problem can be formulated as a constrained combinatorial optimization task that aims to assign one of 1008 distinct PCI values to each cell while minimizing interference and ensuring identifier uniqueness across neighboring cells [10]. The key challenges include mod-3 interference, which occurs when neighboring cells share the same modulo-3 value, as well as collisions, where adjacent cells have the same PCI, and confusions, where non-adjacent cells with common neighbors receive identical PCIs. This problem is NP-hard [11], and its complexity grows significantly with network size, making efficient optimization essential.

This paper addresses the PCI assignment problem by integrating discrete and continuous optimization techniques. Conventional methods, such as genetic algorithms [12] and local search based on binary quadratic programming [9], operate solely in the discrete domain, disregarding valuable first-order derivative information. To overcome these limitations, we propose a novel Relaxed Gradient Projection (RGP) method. Our approach reformulates the discrete constraints into a continuous Cartesian product of probability simplexes, enabling the use of gradient-based optimization. This relaxation improves convergence and solution quality while preserving the multi-objective nature of the problem. In summary, our main contributions are as follows:

- **Multi-objective joint modeling:** We formulate the PCI assignment problem as a unified optimization model that simultaneously minimizes mod-3 interference, collisions, and confusions, capturing their intricate interplay and establishing a suitable framework for PCI assignment.
- **Relaxed gradient projection for optimization:** We relax the discrete multi-objective binary quadratic programming problem into a continuous optimization problem on the Cartesian product of probability simplexes. This relaxation allows efficient gradient-based updates, followed by a rounding procedure to recover feasible PCI assignments.
- **Numerical validation:** Extensive evaluations on real-

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world 5G network data demonstrate that RGP significantly reduces interference while effectively handling collisions and confusions.

Notations. We use \mathbb{Z} and \mathbb{R} to denote the set of integers and real numbers, respectively. Moreover, \mathbb{Z}_k is the set integers ranging from 0 to $k - 1$, \mathbb{R}^N is the set of N -dimensional real-valued vectors, and \mathbb{Z}_k^N is the set of N -dimensional integer vectors with entries from \mathbb{Z}_k . For $a, b \in \mathbb{Z}$ and $k \in \mathbb{Z}_+$, we write $a \equiv b \pmod{k}$ if $a - b = nk$ for some $n \in \mathbb{Z}$. The mod k value of a , denoted by $m_k(a)$, is the unique integer $r \in \mathbb{Z}_k$ such that $a \equiv r \pmod{k}$. We use lowercase and uppercase boldface letters to denote column vectors and matrices, respectively. The notation $[\mathbf{A}]_{i,j}$ represents the element in the i -th row and j -th column of a matrix \mathbf{A} . The transpose of a matrix \mathbf{A} is denoted by \mathbf{A}^T . The trace of a matrix \mathbf{A} is denoted by $\text{Tr}(\mathbf{A})$. The notation $\langle \cdot, \cdot \rangle$ represents the generalized inner product. The function $\mathbb{1}_{\{A\}}$ serves as the indicator function for event A . The cardinality of a set \mathcal{S} is denoted by $|\mathcal{S}|$. The projection operator onto a set \mathcal{S} is denoted by $\mathcal{P}_{\mathcal{S}}(\cdot)$.

II. RELATED WORK

Existing methods for PCI assignment mainly fall into three categories: graph coloring methods [13]–[19], heuristic methods [12], [20], [21], and discrete optimization-based methods [9], [22].

A. Graph Coloring Methods

Graph coloring techniques model a wireless network as a graph, where nodes represent cells and edges indicate potential collisions or confusions. The goal is to assign distinct colors to adjacent nodes to mitigate these issues. Bandh et al. [13] introduced a greedy graph coloring (GGC) method for PCI assignment, which was later improved by Pratap et al. [19] through a randomized approach, enhancing efficiency. However, these methods are not directly applicable to minimizing mod-3 interference due to the complexities introduced by modulo-3 operations.

B. Heuristic Methods

Heuristic algorithms, particularly genetic algorithms, have been explored to address mod-3 interference in the PCI assignment. Li et al. [12] and Shen et al. [20] proposed genetic algorithm-based methods that incorporate mod-3 interference into their fitness functions to address mod-3 interference. However, these approaches lack theoretical guarantees on solution quality and computational complexity, limiting their reliability in large-scale deployments.

C. Discrete Optimization Methods

Discrete optimization methods formulate PCI assignment as a challenging discrete combinatorial optimization problem. Due to its NP-hard nature, most discrete approaches struggle to find high-quality solutions efficiently. As a result, greedy and local search techniques are commonly used, often yielding suboptimal assignments. Gui et al. [9] proposed a greedy local

System Model

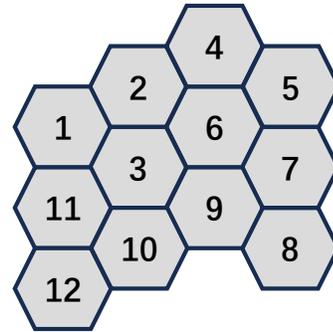


Fig. 1. Illustration of a 5G wireless network comprising $N = 12$ co-frequency cells encountering various challenges arising from improper PCI assignment. Two cells are considered neighboring cells if they share an edge. For example, $(1, 11) \in \mathcal{E}_{\text{neigh}}^{(1)}$ indicates a collision between cell 1 and cell 11 when they share the same PCI value. Moreover, with $(3, 6), (7, 6) \in \mathcal{E}_{\text{neigh}}^{(1)}$, this implies $(3, 7) \in \mathcal{E}_{\text{neigh}}^{(2)}$, where confusion arises when cell 3 and cell 7 are assigned the same PCI. When cell 4 and 8 share the same mod-3 PCI value, mod-3 interference occurs with a magnitude of $[\mathbf{W}_{\text{inter}}]_{4,8}$.

search algorithm to solve the binary quadratic programming (BQP) formulation of PCI assignment, while Fairbrother et al. [22] modeled the problem as a two-level graph partitioning task and developed a mixed-integer programming approach. Both studies introduced weighting schemes to prioritize minimizing mod-3 interference, collisions, and confusions. However, these methods require solving complex discrete optimization problems, making them computationally expensive as network size increases.

III. SYSTEM MODEL AND MULTI-OBJECTIVE MODELING

A. System Model

In a 5G network, each cell is assigned a unique PCI that identifies it within the network. The goal of PCI assignment is to minimize collisions, confusions, and mod-3 interference between cells. The illustration of a 5G wireless network is shown in Fig. 1. To model this problem, we define two graphs representing different relationships: the neighborhood graph and the interference graph.

The first-order neighborhood graph, $\mathcal{G}_{\text{neigh}}^{(1)} = (\mathcal{V}, \mathcal{E}_{\text{neigh}}^{(1)})$, represents direct neighboring relationship between cells. Each vertex $i \in \mathcal{V}$ corresponds to a cell, and an edge $(i, j) \in \mathcal{E}_{\text{neigh}}^{(1)}$ exists if the signal from cell j can be received by devices connected to cell i . The adjacency matrix $\mathbf{W}_{\text{neigh}}^{(1)}$ is defined as

$$[\mathbf{W}_{\text{neigh}}^{(1)}]_{i,j} = \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E}_{\text{neigh}}^{(1)}, \\ 0, & \text{otherwise.} \end{cases}$$

To capture indirect interactions, we define the second-order neighborhood graph, $\mathcal{G}_{\text{neigh}}^{(2)} = (\mathcal{V}, \mathcal{E}_{\text{neigh}}^{(2)})$, where an edge

$(i, j) \in \mathcal{E}_{\text{neigh}}^{(2)}$ exists if there is an intermediate cell k such that $(i, k), (j, k) \in \mathcal{E}_{\text{neigh}}^{(1)}$. The adjacency matrix $\mathbf{W}_{\text{neigh}}^{(2)}$ is given by

$$[\mathbf{W}_{\text{neigh}}^{(2)}]_{i,j} = \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E}_{\text{neigh}}^{(2)} \\ 0, & \text{otherwise.} \end{cases}$$

These graphs are crucial for identifying potential PCI collisions and confusions during handovers and cell re-selections.

Beyond neighborhood relationships, we define the interference graph, $\mathcal{G}_{\text{inter}} = (\mathcal{V}, \mathcal{E}_{\text{inter}})$, to model interference caused by cells sharing the same PCI modulo values. The adjacency matrix $\mathbf{W}_{\text{inter}}$ encodes interference weights, where each entry $[\mathbf{W}_{\text{inter}}]_{i,j}$ quantifies the level of interference between cells i and j . This graph provides a structural foundation for optimizing PCI assignments while mitigating interference.

B. Multi-objective Modeling

Each cell $i \in \mathcal{V}$ is assigned a PCI value $\text{PCI}_i \in \mathbb{Z}_{1008}$. The objective of PCI assignment is to simultaneously minimize collisions, confusions, and mod-3 interference, which is defined as follows:

- A *collision* arises when two neighboring cells i and j are assigned the same PCI, i.e., $(i, j) \in \mathcal{E}_{\text{neigh}}^{(1)}$ and $\text{PCI}_i = \text{PCI}_j$. Collisions prevent the UE from distinguishing the primary serving cells, leading to connection failure.
- A *confusion* arises when two cells i and j , share a common neighbor and are assigned the same PCI, i.e., $(i, j) \in \mathcal{E}_{\text{neigh}}^{(2)}$ and $\text{PCI}_i = \text{PCI}_j$. Confusions disrupt cell handovers of UE, leading to service interruptions.
- *Mod-3 interference* occurs when two cells have PCI values that are congruent modulo 3, i.e., $\text{PCI}_i \equiv \text{PCI}_j \pmod{3}$. The magnitude of mod-3 interference between two cells is given by $[\mathbf{W}_{\text{inter}}]_{i,j}$. Such interference can cause potential resource conflicts and signal degradation, negatively impacting network performance.

The problem can be formulated as a multi-objective optimization problem:

$$\min_{\text{PCI}=(\text{PCI}_1, \dots, \text{PCI}_{|\mathcal{V}|})} \left(\frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{neigh}}^{(1)}]_{i,j} \cdot \mathbb{1}_{\{\text{PCI}_i = \text{PCI}_j\}}, \quad (1a) \right.$$

$$\frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{neigh}}^{(2)}]_{i,j} \cdot \mathbb{1}_{\{\text{PCI}_i = \text{PCI}_j\}}, \quad (1b)$$

$$\left. \frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{inter}}]_{i,j} \cdot \mathbb{1}_{\{\text{PCI}_i \equiv \text{PCI}_j \pmod{3}\}} \right), \quad (1c)$$

$$\text{s.t.} \quad \text{PCI}_i \in \mathbb{Z}_{1008}, \forall i \in \mathcal{V}. \quad (1d)$$

This is a multi-objective optimization problem where the objectives are to minimize the collisions (1a), the confusions (1b), and the mod-3 interference (1c). The constraints include the PCI range requirement (1d).

IV. A RELAXED GRADIENT PROJECTION SCHEME FOR PCI ASSIGNMENT

In this section, we propose the RPG scheme for solving the multi-objective optimization problem (1). We first reformulate (1) as a BQP problem, similar to [9]. We then relax this discrete BQP into a continuous optimization problem defined on the Cartesian product of probability simplexes. Subsequently, we apply the gradient projection algorithm to efficiently solve the relaxed problem. Finally, we round the continuous solution to obtain the final PCI assignment.

A. Binary Quadratic Programming and Probability Simplex Relaxation

We first apply the one-hot encoding scheme to represent PCI values, where each cell $i \in \mathcal{V}$ is associated with a unit vector $\mathbf{x}_i = \mathbf{e}_{\text{PCI}_i}$, and \mathbf{e}_ℓ is a standard basis vector in \mathbb{R}^{1008} . Stacking these vectors into a matrix $\mathbf{X} = (\mathbf{x}_1, \dots, \mathbf{x}_{|\mathcal{V}|}) \in \{0, 1\}^{1008 \times |\mathcal{V}|}$, we reformulate the collision and confusion objectives as quadratic terms given by

$$\begin{aligned} \frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{neigh}}^{(1)}]_{i,j} \cdot \mathbb{1}_{\{\text{PCI}_i = \text{PCI}_j\}} &= \frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{neigh}}^{(1)}]_{i,j} \mathbf{x}_i^T \mathbf{x}_j \\ &= \frac{1}{2} \text{Tr}(\mathbf{X} \mathbf{W}_{\text{neigh}}^{(1)} \mathbf{X}^T), \\ \frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{neigh}}^{(2)}]_{i,j} \cdot \mathbb{1}_{\{\text{PCI}_i = \text{PCI}_j\}} &= \frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{neigh}}^{(2)}]_{i,j} \mathbf{x}_i^T \mathbf{x}_j \\ &= \frac{1}{2} \text{Tr}(\mathbf{X} \mathbf{W}_{\text{neigh}}^{(2)} \mathbf{X}^T). \end{aligned}$$

Similarly, we express mod-3 interference using an auxiliary matrix $\mathbf{L}^{(3)} = [\mathbf{l}_1^{(3)}, \dots, \mathbf{l}_3^{(3)}] \in \mathbb{R}^{1008 \times 3}$ given by.

$$\mathbf{L}^{(3)} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & \dots & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & \dots & 0 & 0 & 1 \end{pmatrix}^T,$$

where each column represents the cyclic structure of PCI indices mod 3. Then, the mod-3 interference objective can be rewritten as

$$\begin{aligned} &\frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{inter}}]_{i,j} \cdot \mathbb{1}_{\{\text{PCI}_i \equiv \text{PCI}_j \pmod{3}\}} \\ &= \frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{inter}}]_{i,j} \sum_{k=0}^2 \mathbb{1}_{\{m_3(\text{PCI}_i) = m_3(\text{PCI}_j) = k\}} \\ &= \frac{1}{2} \sum_{i,j=1}^{|\mathcal{V}|} [\mathbf{W}_{\text{inter}}]_{i,j} \sum_{k=1}^3 \mathbf{x}_i^T \mathbf{l}_k^{(3)} \cdot \mathbf{x}_j^T \mathbf{l}_k^{(3)} \\ &= \frac{1}{2} \text{Tr}(\mathbf{L}^{(3)T} \mathbf{X} \mathbf{W}_{\text{inter}} \mathbf{X}^T \mathbf{L}^{(3)}). \end{aligned}$$

Thus, problem (1) is reformulated as a multi-objective BQP as

$$\begin{aligned} \min_{\mathbf{X}=(\mathbf{x}_1, \dots, \mathbf{x}_{|\mathcal{V}|})} & \left(\frac{1}{2} \text{Tr} \left(\mathbf{X} \mathbf{W}_{\text{neigh}}^{(1)} \mathbf{X}^T \right), \right. \\ & \frac{1}{2} \text{Tr} \left(\mathbf{X} \mathbf{W}_{\text{neigh}}^{(2)} \mathbf{X}^T \right), \\ & \left. \frac{1}{2} \text{Tr} \left(\mathbf{L}^{(3)T} \mathbf{X} \mathbf{W}_{\text{inter}} \mathbf{X}^T \mathbf{L}^{(3)} \right) \right), \quad (2) \\ \text{s.t.} & \quad \mathbf{x}_i \in \{\mathbf{e}_0, \dots, \mathbf{e}_{1007}\}, \forall i \in \mathcal{V}. \end{aligned}$$

To solve this multi-objective problem efficiently, we use a weighted summation approach leading to

$$\begin{aligned} \min_{\mathbf{X}=(\mathbf{x}_1, \dots, \mathbf{x}_{|\mathcal{V}|})} & \frac{\lambda_1}{2} \text{Tr} \left(\mathbf{X} \mathbf{W}_{\text{neigh}}^{(1)} \mathbf{X}^T \right) + \frac{\lambda_2}{2} \text{Tr} \left(\mathbf{X} \mathbf{W}_{\text{neigh}}^{(2)} \mathbf{X}^T \right) \\ & + \frac{\lambda_3}{2} \text{Tr} \left(\mathbf{L}^{(3)T} \mathbf{X} \mathbf{W}_{\text{inter}} \mathbf{X}^T \mathbf{L}^{(3)} \right) \quad (3) \\ \text{s.t.} & \quad \mathbf{x}_i \in \{\mathbf{e}_0, \dots, \mathbf{e}_{1007}\}, \forall i \in \mathcal{V}, \end{aligned}$$

where $\lambda_1, \lambda_2, \lambda_3$ are the weight coefficient for collision, confusion, and mod-3 interference, respectively.

Since the constraint $\mathbf{x}_i \in \{\mathbf{e}_0, \dots, \mathbf{e}_{1007}\}$ is discrete and nonconvex, we relax the constraint into its convex envelop, the probability simplex $\Delta_{1008} = \{\mathbf{x} \in \mathbb{R}^{1008} \mid \sum_{i=1}^{1008} \mathbf{x}_i = 1, \mathbf{x} \geq 0\}$. The problem is then transformed into

$$\begin{aligned} \min_{\mathbf{X}} & \quad F(\mathbf{X}) \quad (4) \\ \text{s.t.} & \quad \mathbf{X} \in \Delta_{1008}^{|\mathcal{V}|}, \end{aligned}$$

where $\Delta_{1008}^{|\mathcal{V}|}$ is the Cartesian product of $|\mathcal{V}|$ 1008-dimensional probability simplexes.

B. Gradient Projection

Problem (4) involves optimizing a non-convex objective function over a convex constraint set. A natural approach is the gradient projection method [23]–[25], which iteratively updates the solution as

$$\begin{aligned} \mathbf{X}^{(m+1)} &= \mathcal{P}_{\Delta_{1008}^{|\mathcal{V}|}} \left(\mathbf{X}^{(m)} - t_m \nabla F(\mathbf{X}^{(m)}) \right) \\ &= \arg \min_{\mathbf{X} \in \Delta_{1008}^{|\mathcal{V}|}} \left\{ F(\mathbf{X}^{(m)}) + \langle \nabla F(\mathbf{X}^{(m)}), \mathbf{X} \right. \\ & \quad \left. - \mathbf{X}^{(m)} \rangle + \frac{1}{2t_m} \underbrace{\|\mathbf{X} - \mathbf{X}^{(m)}\|_F^2}_{\text{Euclidean distance}} \right\}, \quad (5) \end{aligned}$$

where t_m is the step-size at iteration m .

The gradient of $F(\mathbf{X})$ is given by

$$\begin{aligned} \nabla F(\mathbf{X}) &= \mathbf{X} (\lambda_1 \mathbf{W}_{\text{neigh}}^{(1)} + \lambda_2 \mathbf{W}_{\text{neigh}}^{(2)}) + \lambda_3 \mathbf{L}^{(3)} \mathbf{L}^{(3)T} \mathbf{X} \mathbf{W}_{\text{inter}} \\ &:= \mathbf{X} \mathbf{A} + \mathbf{B} \mathbf{X} \mathbf{W}_{\text{inter}} \end{aligned}$$

Iterating this update yields a stationary point of problem (4), providing a continuous solution $\tilde{\mathbf{X}}$. To obtain a discrete PCI assignment \mathbf{PCI} , we apply a simple rounding step, shown as

$$\widetilde{\mathbf{PCI}}_j = \arg \max_i [\tilde{\mathbf{X}}]_{ij}, \quad \forall j \in \mathcal{V}.$$

This rounding step assigns each cell the PCI value corresponding to its highest probability in $\tilde{\mathbf{X}}$, ensuring a valid discrete solution.

C. Convergence Analysis

Gradient projection is known to converge to a stationary point for L -smooth objective functions under a constant step-size [26], [27]. Applying this result to problem (4), we establish the following convergence theorem.

Theorem 1 (Convergence Rate of Gradient Projection, Theorem 2.6.1 [26]). *The sequence $\{\mathbf{X}^{(m)}\}$ generated by the gradient projection method with step-size $t_m = \frac{1}{\|\mathbf{A}\| + \|\mathbf{B}\| \|\mathbf{W}_{\text{inter}}\|}$ converges to a stationary point of problem (4) at a rate of $\mathcal{O}(\frac{1}{\epsilon^2})$ with respect to the stopping criterion $T(\mathbf{X}) \leq \epsilon$, where*

$$T(\mathbf{X}) = \left\| \mathcal{P}_{\Delta_{1008}^{|\mathcal{V}|}} \left(\mathbf{X} - \frac{\mathbf{X} \mathbf{A} + \mathbf{B} \mathbf{X} \mathbf{W}_{\text{inter}}}{\|\mathbf{A}\| + \|\mathbf{B}\| \|\mathbf{W}_{\text{inter}}\|} \right) - \mathbf{X} \right\|. \quad (6)$$

Proof. To establish convergence, it suffices to show that $F(\mathbf{X})$ is $\|\mathbf{A}\| + \|\mathbf{B}\| \|\mathbf{W}_{\text{inter}}\|$ -smooth. This follows from

$$\begin{aligned} \|\nabla F(\mathbf{X}) - \nabla F(\mathbf{Y})\| &= \|(\mathbf{X} - \mathbf{Y}) \mathbf{A} + \mathbf{B}(\mathbf{X} - \mathbf{Y}) \mathbf{W}_{\text{inter}}\| \\ &\leq \|\mathbf{X} - \mathbf{Y}\| (\|\mathbf{A}\| + \|\mathbf{B}\| \|\mathbf{W}_{\text{inter}}\|), \end{aligned}$$

which completes the proof. \square

D. Overall Implementation of the RPG Algorithm

To avoid non-differentiable initial points at the boundary of the simplex Δ_k^N , we adopt a random initialization strategy for the initial point $\mathbf{X}^{(0)} = [\mathbf{x}_0^{(0)}, \dots, \mathbf{x}_N^{(0)}]$. Specifically, for each column $\mathbf{x}_j^{(0)}$, we have

Random Initialization :

Step 1: Generate $\mathbf{z} \stackrel{i.i.d.}{\sim} U^{k \times 1} [0, 1]$, (7)

Step 2: Normalize $\mathbf{x}_j^{(0)} = \frac{\mathbf{z}}{\|\mathbf{z}\|_1}$.

The complete implementation of the RGP algorithm for PCI assignment is presented in Algorithm 1.

V. NUMERICAL EVALUATIONS

We conduct numerical evaluations to evaluate the performance and efficiency of our PCI assignment approach against state-of-the-art baselines. All evaluations are implemented in Python and executed on a machine with an Intel i9-12900K CPU and an NVIDIA GeForce RTX 3090 GPU.

A. List of Baseline Methods

We compare our proposed Relaxed Gradient Projection (RGP) method with three representative approaches from different categories, which are described below.

Greedy Graph Coloring Method (GGC) [13]: This method employs a greedy coloring algorithm to mitigate collisions and confusions. We use the implementation provided in [28] in the simulation.

Heuristic Method (Genetic) [12]: A heuristic-based approach with a population size of 30, 200 iterations, and crossover, mutation, and selection probabilities set to 0.77, 0.3, and 0.8, respectively.

Discrete Optimization Method (BQP) [9]: A discrete optimization approach addressing collisions, confusions, and

Algorithm 1: Proposed RGP Method for PCI assignment

Data: Adjacency matrices $\mathbf{W}_{\text{neigh}}^{(1)}$, $\mathbf{W}_{\text{neigh}}^{(2)}$, and $\mathbf{W}_{\text{inter}}$ for graphs $\mathcal{G}_{\text{neigh}}^{(1)}$, $\mathcal{G}_{\text{neigh}}^{(2)}$, and $\mathcal{G}_{\text{inter}}$; weights $\lambda_1, \lambda_2, \lambda_3$ for collision, confusion, and mod-3 interference; tolerance parameter $\epsilon > 0$.

Result: Solution **PCI**

```

// Random Initialization
for  $j = 1$  to  $|\mathcal{V}|$  independently do
  | Generate  $\mathbf{x}_j^0$  by Random Initialization (7)
Set  $\mathbf{X}^{(0)} = [\mathbf{x}_1^{(0)}, \dots, \mathbf{x}_{|\mathcal{V}|}^{(0)}]$ ;
Set  $\mathbf{A} = \lambda_1 \mathbf{W}_{\text{neigh}}^{(1)} + \lambda_2 \mathbf{W}_{\text{neigh}}^{(2)}$ ,  $\mathbf{B} = \lambda_3 \mathbf{L}^{(3)} \mathbf{L}^{(3)T}$ ;

// RGP Iteration
Set  $m = 0$ ;
repeat
  | Set
    |  $\mathbf{X}^{(m+1)} = \mathcal{P}_{\Delta_{1008}^{|\mathcal{V}|}}(\mathbf{X}^{(m)} - \frac{\mathbf{X}^{(m)} \mathbf{A} + \mathbf{B} \mathbf{X}^{(m)} \mathbf{W}_{\text{inter}}}{\|\mathbf{A}\| + \|\mathbf{B}\| \|\mathbf{W}_{\text{inter}}\|})$ 
    | Set  $m = m + 1$ 
until  $T(\mathbf{X}^{(m)}) \leq \epsilon$ ;

// Rounding
Set  $\tilde{\mathbf{X}} = \mathbf{X}^{(m)}$ 
for  $j = 1$  to  $|\mathcal{V}|$  independently do
  | Set  $\widetilde{\text{PCI}}_j = \arg \max_i [\tilde{\mathbf{X}}]_{ij}$ .
return  $\widetilde{\text{PCI}}$ 

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mod-3 interference. We adopt the hyperparameters from the *1-1-1 Greedy* experiment in [9].

B. Evaluation Set-up

We conduct numerical evaluations using a real-world PCI assignment dataset from Beijing, featuring antennas deployed in a SISO system. The dataset captures mod-3 interference, collisions, and confusions in a live downlink network with 947 cells, covering a dense urban area of 53.09 km². Measurement reports (MR) collected from UE provide network status and performance data, forming the graphs $\mathcal{G}_{\text{neigh}}^{(1)}$, $\mathcal{G}_{\text{neigh}}^{(2)}$, and $\mathcal{G}_{\text{inter}}$.

For collision detection, a pair $(i, j) \in \mathcal{E}_{\text{neigh}}^{(1)}$ is included if both cells share the same frequency and MR data indicates that j is a neighbor of i . In the case of confusion, a pair $(i, j) \in \mathcal{E}_{\text{neigh}}^{(2)}$ is included if they share the same frequency and both are reported as neighbors of another cell k . For the interference matrix, if cell j is a neighboring cell sharing the same frequency as cell i , the entry $[\mathbf{W}]_{i,j}$ represents the number of MRs between these two cells. To evaluate scalability, we test our method on datasets with different numbers of cells, and $|\mathcal{V}| = 53, 626, 947$. The tolerance threshold for gradient projection is set to $\epsilon = 10^{-8}$. The weights $\lambda_1, \lambda_2, \lambda_3$ are all set to 1.

TABLE I
THE EVALUATION RESULTS ON THE REAL-WORLD DATA
WITH $|\mathcal{V}| = 53$.

Models	Coll.	Conf.	Mod-3.	Time (s)
GGC [13], [28]	0	0	2650108	0.012
Genetic [12]	1	1	136823	5
BQP [9]	0	0	127219	71
RGP	0	2	116609	41

C. Evaluation Results

Tables I, II, and III summarize the evaluational results.¹ The results show that RGP consistently achieves the lowest mod-3 interference while maintaining competitive performance in minimizing collisions and confusions.

For the smallest instance with size $|\mathcal{V}| = 53$, RGP reduces mod-3 interference by 8.3% compared to the best-performing baseline BQP while running in less time. In larger instances with size $|\mathcal{V}| = 626$ and $|\mathcal{V}| = 947$, RGP outperforms Genetic in minimizing mod-3 interference by 20.8% and 29.0%, respectively. GGC achieves zero collisions and confusions, but performs poorly in addressing mod-3 interference, yielding significantly higher values than all other methods. Genetic produces substantially more collisions and confusions while also exhibiting higher mod-3 interference. BQP fails to return a solution within the time limit for large-scale instances, making it impractical for real-world applications. Overall, RGP demonstrates superior effectiveness in balancing interference minimization and computational efficiency, making it a promising solution for large-scale PCI optimization.

VI. CONCLUSION

This paper introduces a novel RGP method for solving the PCI assignment in 5G networks. By reformulating the discrete optimization problem into a continuous framework and applying gradient projection techniques, RGP effectively minimizes collisions, confusions, and mod-3 interference. Evaluations on real-world 5G network data demonstrate the superiority of RGP over traditional graph coloring, heuristic, and discrete optimization-based methods. RGP consistently achieves lower mod-3 interference while maintaining superior performance in reducing collisions and confusions. Moreover, RGP scales efficiently to large network instances, demonstrating its practicality for real-world deployment. Future work may focus on extending this framework to incorporate additional network constraints and further enhance computational efficiency for ultra-dense 5G networks.

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¹In the tables, 'Coll.' represents the number of collisions, 'Conf.' denotes the number of confusions, 'Mod-3' refers to the mod-3 interference value, and 'Time' indicates the runtime of each method.

TABLE II
THE EVALUATION RESULTS ON THE REAL-WORLD DATA
WITH $|\mathcal{V}| = 626$.

Models	Coll.	Conf.	Mod-3.	Time (s)
GGC [13], [28]	0	0	28148512	0.027
Genetic [12]	22	42	5857148	503
BQP [9]*	*	*	*	*
RGP	22	50	4641011	4114

*: BQP fails to return a solution in one day.

TABLE III
THE EVALUATION RESULTS ON THE REAL-WORLD DATA WITH
 $|\mathcal{V}| = 947$.

Models	Coll.	Conf.	Mod-3.	Time (s)
GGC [13], [28]	0	0	40409198	0.057
Genetic [12]	39	80	10582454	1204
BQP [9]*	*	*	*	*
RGP	31	64	7511334	6655

*: BQP fails to return a solution in two weeks of time.

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